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Sunspots Activity Cycles Using Linear and Non-Linear Technique

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ABSTRACT

Sun and its activities have been the unique importance to understand the space weather. Especially the sunspots at solar maxima and minima, which are responsible for space weather variability are the main concern for the scientific society of the world. Better modeling for solar maxima and minima can be useful to understand the space weather precisely. The research work in this paper provides the trend analysis for each sunspot cycle (1-24), in which 24th cycle is in continuation. The increasing or decreasing trend of sunspot cycles provides the information regarding increase and decrease of the solar dynamics. We have compared the trend of 1st to 24th sunspot cycles of different duration under the significance test. The minimum and maximum sunspots in each cycle are discussed using the trend analysis which is useful to study the space weather.

Keywords: Sunspot cycle, trend analysis, solar maxima, and minima.

Introduction

"The sunspot is the main solar activity on which other activities of the sun are associated. Sunspots have very well defined the cycle of average 11.1 years along with smallest and greatest cycle of 9 years and 14 years respectively.[1, 3] Components of a Time Series There are four components of a time series: the trend, the cyclical variation, the seasonal variation, and the irregular variation.[2] Secular Trend the long-term trends of sales, employment, stock prices, and other business and economic series follow various patterns. Some move steadily upward, others decline, and still, others stay the same over time.[6]

Cyclical Variation the second component of a time series is cyclical variation. A typical business cycle consists of a period of prosperity followed by periods of recession, depression, and then recovery with no fixed duration of the cycle.[5] There are sizable fluctuations unfolding over more than one year in time above and below the secular trend. In a recession, for example, employment, production, the S and P/TSX Composite Index, and many other businesses and economic series are below the long-term trend lines. Conversely, in periods of prosperity, they are above their long-term trend lines.[7, 8]

Irregular Variation Many analysts prefer to subdivide the irregular variation into episodic and residual variations. Episodic fluctuations are unpredictable, but they can be identified. The initial impact on the economy of a major strike or a war can be identified, but a strike or war cannot be predicted[8] after the episodic fluctuations have been removed, the remaining variation is called the residual variation. The residual fluctuations, often called chance fluctuations, are unpredictable, and they cannot be identified. Of course, neither episodic nor residual variation can be projected into the future.[4, 9] The analysis may also be useful to observe the solar impact on earth's climate for any specific location, the significant relation is found between sunspot cycle and ENSO.[10]

Materials and Methods

The study is performed on time series; data of sunspots of long period from 1754 to 2015. Data are collected from the world data center. We are evaluated the 24th cycles Data by using MINITAB and EXCEL software. In this research paper, trend analysis technique has utilized. The nonlinear trend model is more significant as compared to the linear trend model.

The trend analysis is applied to fit a general trend model to data and offers an assessment. Trend analysis consists of linear trend model and nonlinear trend models.

Linear analysis of solar Activity cycles

The linear trend model as recommended by.[9,10] described as follows:

$$Yx = \beta_o + \beta_1 x \qquad (2.1)$$

Where β_o and β_1 are the parameters mentioned below

$$\beta_o = \frac{n\sum y_i x_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2} \qquad (2.1\ a)$$

$$\beta_1 = \frac{\sum y_i \sum x_i^2 - \sum x_i^2 \sum x_i y_i}{n\sum x_i^2 - (\sum x_i)^2} \qquad (2.1\ b)$$

Where β_1 is represented by the average change from one period to another period?

Non-Linear analysis of solar Activity cycles

The quadratic model was used for this study as this model was also applied by.[9] the model equation is given by:

$$Yx = \beta o + \beta_1 x + \beta_2 x^2 \qquad (2.2)$$

The parameters of the above equations are as under

$$\beta_1 = \frac{SxySx^2x^2 - Sx^2ySxx^2}{SxxSx^2x^2 - (Sxx^2)} \qquad (2.2\ a)$$

$$\beta_2 = \frac{Sx^2ySxx - SxySxx^2}{SxxSx^2x^2 - (Sxx^2)^2} \qquad (2.2\ b)$$

To obtain the above parameters the value of Sxx, Sxy, S_{xx^2}, $S_{x^2x^2}$ and S_{x^2y} can be estimated using the following conditions.

$$Sxx = \frac{(\sum x_i - \overline{x})}{n} = \frac{\sum x_i^2}{n} - \overline{x^2}$$

$$Sxy = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n} = \frac{\sum x_i y_i}{n} - \bar{x}\bar{y}$$

$$S_{xx^2} = \frac{\sum (x_i - \bar{x})(x_i^2 - \bar{x^2})}{n} = \frac{\sum x_i^3}{n} - \bar{x}\bar{x^2}$$

$$S_{x^2x^2} = \frac{\sum (x_i^2 - x^2)^2}{n} = \frac{\sum x_i^4}{n} - \bar{x^2}\bar{x^2}$$

$$S_{x^2y} = \frac{\sum (x_i^2 - \bar{x^2})(Y_i - \bar{Y})}{n} = \frac{\sum x_i^2 y_i}{n} - \bar{x^2}\bar{y} \qquad (2.2\ c)$$

Y**x** is the predicted area and yield, **x** is the time index β0 is the constant of model and β_1 and β_2 is the monthly area and yield change.

Discussion & Recommendations

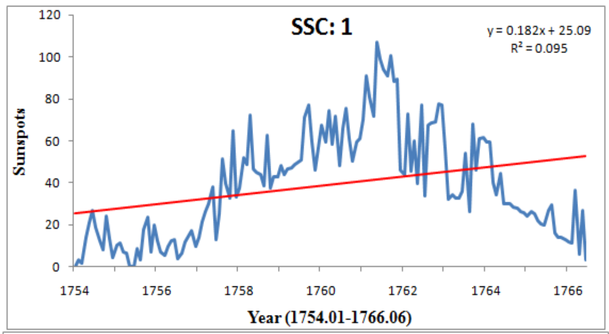
The research work includes the 24 sunspot cycles, the 24th cycle is not completed. We have tried to fit the linear and quadratic model on each cycle. The linear trend represents the rise or fall of the solar dynamics in the respective cycle. Increasing trend shows the long dynamic while decreasing trend indicates the short dynamic within the cycle. Increasing trend has the mean above the trend line in the cycle. The quadratic model represents the significant as compare to the linear trend. Table 1 and 2 depict that the value of R^2 is more significant estimated using the quadratic model. The value of the coefficient of determination's parameters indicates the greater value for the quadratic model for each sunspot cycle as compare to the linear model depicts the tables 1and 2. Some goodness-of-fit tests Mean Absolute Percentage Error (MAPE), Mean Absolute Deviation (MAD) and Mean Standard Deviation (MSD) are applied to verify the significance model. Our analysis shows the least value found for MAPE, MAD, and MSD when fitted the quadratic model on each sunspot cycle. The goodness-of-fit test values found greater for linear trend model fitted on each cycle depicts the tables 1 & 2 (also see Fig. 1 and 2 for comparison). Our analysis shows that the cyclic behavior of solar activity can be better modeled by the quadratic model (non-linear). Figure 3 depicts the significance of the coefficient of determination for quadratic model applied on each sunspots cycle. We have tried to fit the linear and quadratic model on 24th sunspot cycle, although it is not completed yet. Fitting the quadratic model of the 24th sunspot cycle may be useful to forecast the cycle.

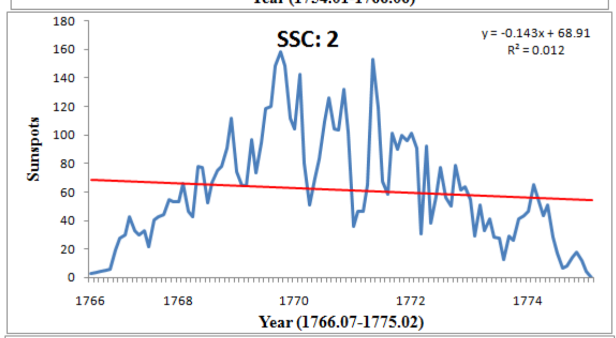
Table:1 summary of linear equation of each sunspots cycles from 1754-2015

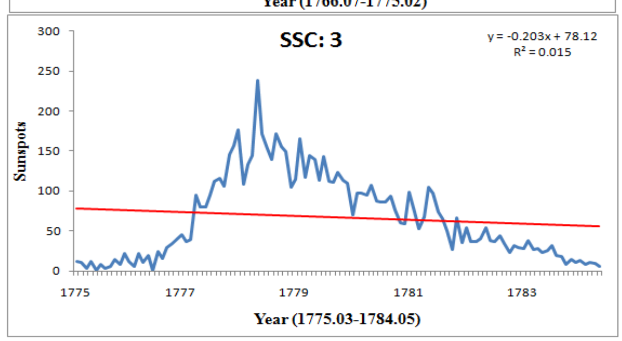
Cycles	Linear Trend				
	Equation	R^2	MAPE	MAD	MSD
1	Yx = 0.182x + 25.09	0.095	119.829	20.540	591.893
2	Yx = -0.143x + 68.91	0.012	149.58	30.49	1427.29
3	Yx = -0.143x + 68.91	0.015	328.52	44.17	283.13
4	Yx= -0.466x + 99.71	0.250	169.65	29.44	1530.98
5	Yx = 0.161x + 19.16	0.094	125.589	14.363	276.130
6	Yx = 0.146x + 4.688	0.156	337.634	11.687	258.943
7	Yx = 0.353x + 10.57	0.256	263.183	18.893	564.206
8	Yx = -0.093x + 72.68	0.004	236.73	40.03	2284.46
9	Yx = -0.058x + 59.73	0.004	225.49	30.79	1539.19
10	Yx = -0.022x + 49.81	0.000	332.64	26.82	1009.32
11	Yx = -0.454x + 85.74	0.152	676.02	34.65	1842.39
12	Yx = -0.167x + 45.41	0.059	518.463	21.796	672.600
13	Yx = -0.290x + 60.32	0.145	385.304	22.466	770.771
14	Yx= 0.043x + 31.30	0.003	433.126	21.567	671.023
15	Yx= 0.026x + 35.94	0.001	734.63	29.19	1209.82
16	Yx= -0.319x + 61.25	0.143	244.329	21.452	667.817
17	Yx= -0.116x + 63.86	0.001	631.04	34.16	1639.06
18	Yx = -0.318x + 96.27	0.040	480.75	45.15	2753.16
19	Yx= -0.445x + 119.3	0.052	1077.74	59.58	4767.94
20	Yx= -0.217x + 74.53	0.057	153.58	31.42	1337.67
21	Yx= -0.436x + 109.8	0.069	217.18	47.83	3013.54
22	Yx= -0.586x + 112.5	0.128	261.52	46.08	2979.72
23	Yx=-0.544x + 94.89	0.305	255.10	28.16	1328.89
24	Yx= 0.960x + 18.11	0.526	52.653	14.568	328.670

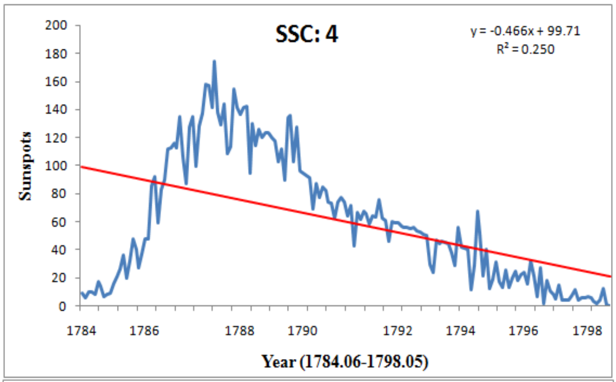
Table:2 Summary of quadratic equation of each sunspots cycles from 1754-2015

Cycles	Quadratic Trend				
	Equation	R2	MAPE	MAD	MSD
1	Yx = -0.011x2 + 1.861x - 17.42	0.626	67.679	12.921	244.512
2	Yx= -0.036x2 + 3.716x + 0.708	0.620	44.264	17.709	549.073
3	Yx= -0.045x2 + 4.858x - 17.20	0.647	129.759	24.034	961.295
4	Yx= -0.012x2 + 1.641x + 39.98	0.587	128.281	24.532	842.487
5	Yx= -0.014x2 + 1.838x - 13.52	0.759	51.9972	6.7435	73.3435
6	Yx= -0.003x2 + 0.753x - 12.60	0.351	248.992	10.975	201.241
7	Yx= -0.011x2 + 1.963x - 26.74	0.607	200.140	13.501	297.615
8	Yx= -0.038x2 + 4.415x - 15.98	0.655	89.909	22.399	791.137
9	Yx= -0.018x2 + 2.758x - 12.09	0.645	73.266	16.889	292.139
10	Yx= -0.019x2 + 2.639x - 11.40	0.710	67.672	14.075	292.139
11	Yx= -0.021x2 + 2.493x + 16.46	0.575	699.256	24.843	922.976
12	Yx= -0.015x2 + 1.882x - 1.383	0.645	286.512	13.041	253.544
13	Yx= -0.014x2 + 1.707x + 14.05	0.600	228.929	15.642	360.947
14	Yx= -0.016x2 + 2.238x - 16.98	0.665	138.323	11.127	225.617
15	Yx= -0.018x2 + 2.638x - 27.18	0.632	285.426	16.633	445.461
16	Yx = -0.020x2 + 2.083x + 14.40	0.677	158.603	12.967	251.279
17	Yx= -0.028x2 + 3.499x - 14.47	0.717	169.704	16.986	468.055
18	Yx= -0.043x2 + 4.857x - 6.393	0.738	241.384	21.811	750.340
19	Yx= -0.047x2 + 5.607x - 9.826	0.684	214.74	33.98	1587.52
20	Yx= -0.019x2 + 2.651x + 4.71	0.716	61.769	16.115	402.585
21	Yx= -0.044x2 + 4.915x + 1.909	0.753	100.965	23.655	798.440
22	Yx= -0.037x2 + 4.117x + 13.81	0.672	177.65	28.23	1120.80
23	Yx= -0.013x2 + 1.596x + 39.23	0.617	262.692	22.726	733.083
24	Yx= -0.030x2 + 3.076x -6.931	0.692	33.027	11.139	213.245





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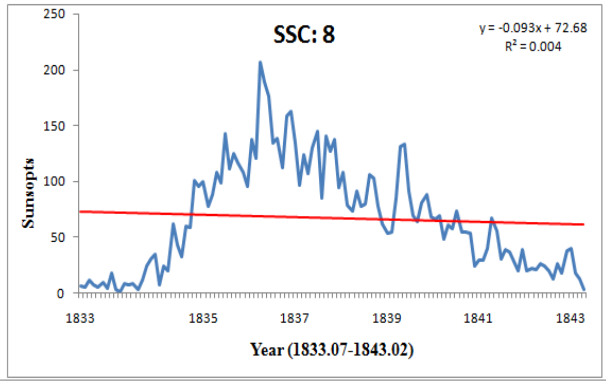
Sunspots
Year (1784.06-1798.05)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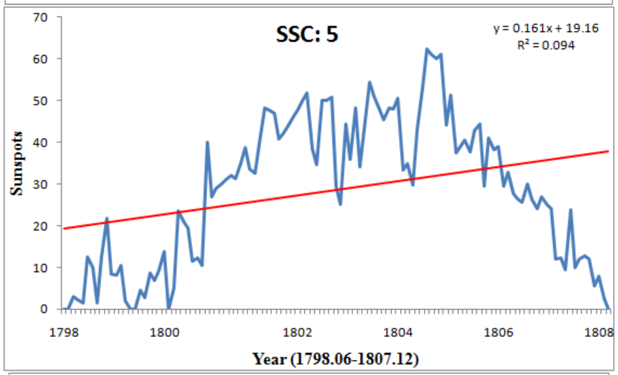
Sunspots
Year (1833.07-1843.02)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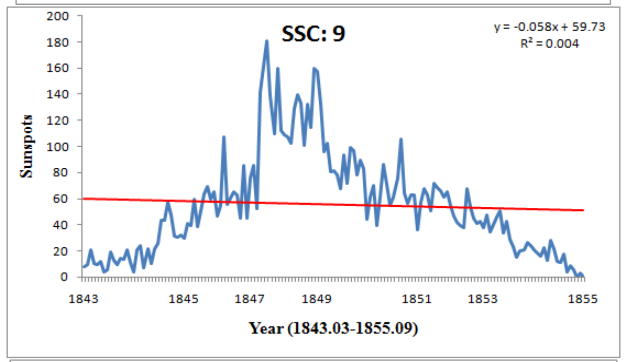
R² = 0.094
Sunspots
Year (1798.06-1807.12)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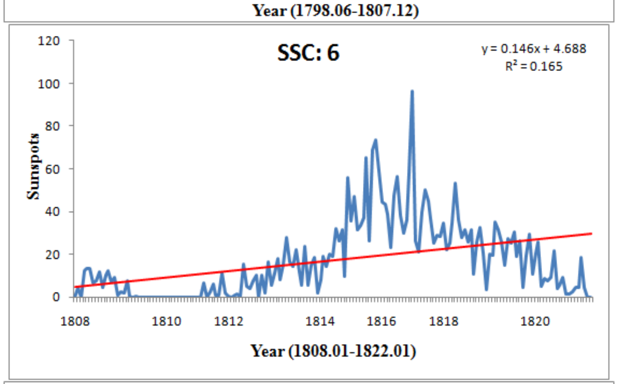
R² = 0.004
Sunspots
Year (1843.03-1855.09)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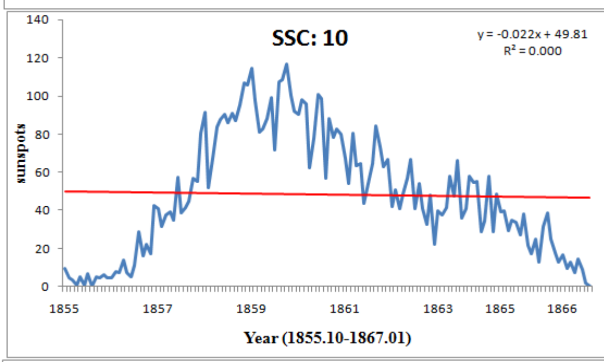
y = 0.146x + 4.688
R² = 0.165
Sunspots
Year (1808.01-1822.01)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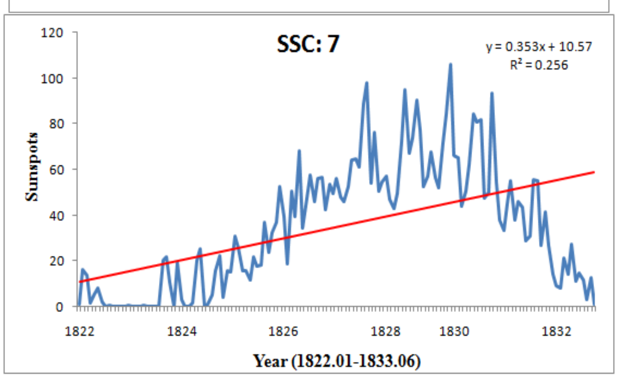
R² = 0.000
sunspots
Year (1855.10-1867.01)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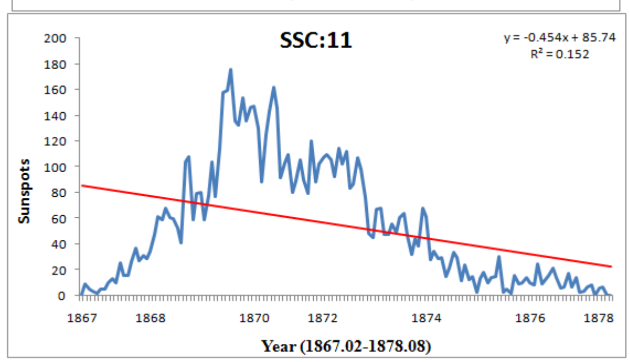
y = 0.353x + 10.57
R² = 0.256
Sunspots
Year (1822.01-1833.06)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y = -0.454x + 85.74
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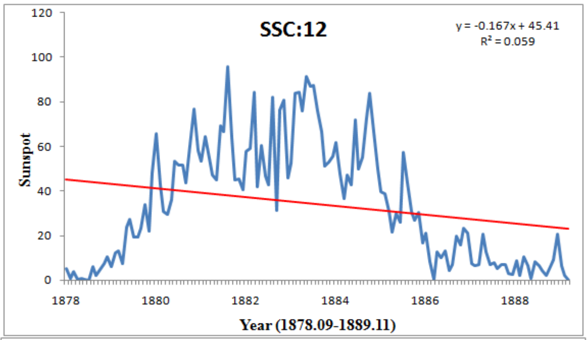
Sunspots
Year (1867.02-1878.08)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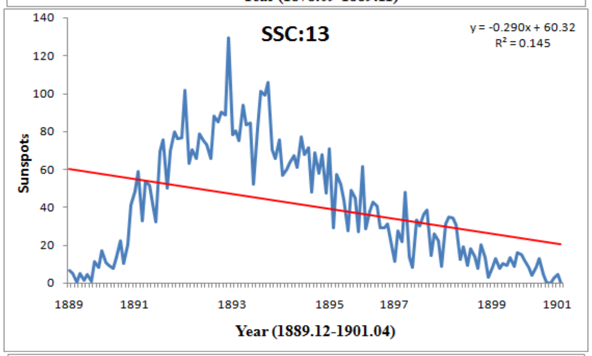
y = -0.167x + 45.41
R² = 0.059
Sunspot
Year (1878.09-1889.11)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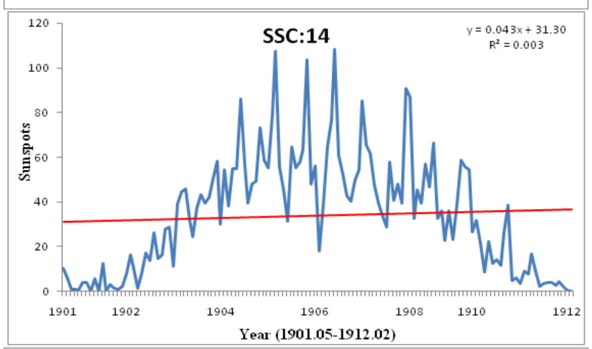
R² = 0.145
Sunspots
Year (1889.12-1901.04)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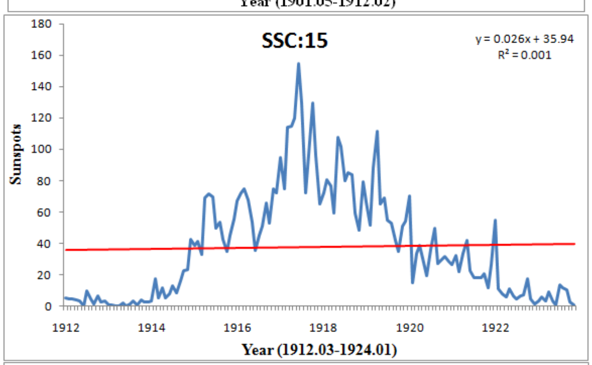
y = 0.043x + 31.30
R² = 0.003
Sunspots
Year (1901.05-1912.02)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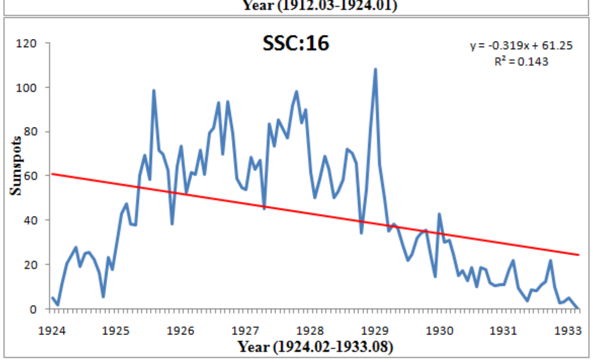
R² = 0.001
Sunspots
Year (1912.03-1924.01)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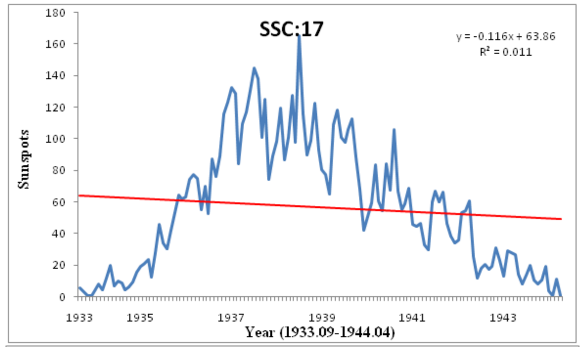
y = -0.319x + 61.25
R² = 0.143
Sunspots
Year (1924.02-1933.08)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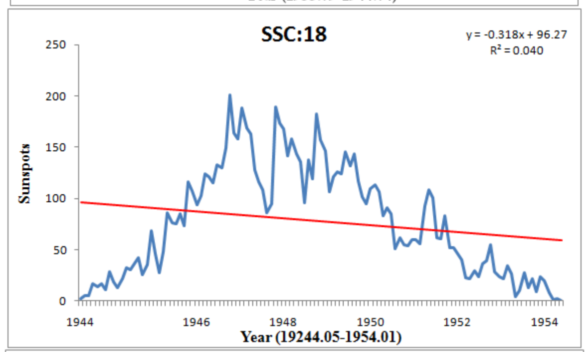
y = -0.116x + 63.86
R² = 0.011
Sunspots
Year (1933.09-1944.04)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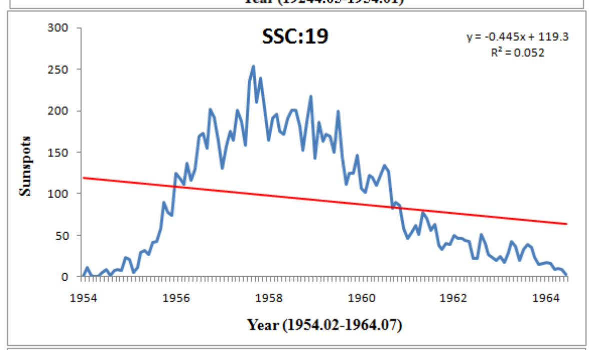
y = -0.318x + 96.27
R² = 0.040
Sunspots
Year (19244.05-1954.01)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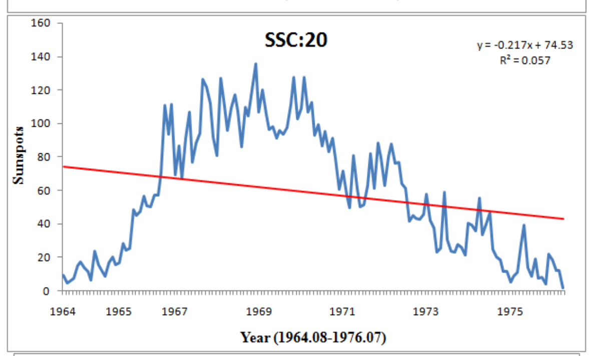
y = -0.445x + 119.3
R² = 0.052
Sunspots
Year (1954.02-1964.07)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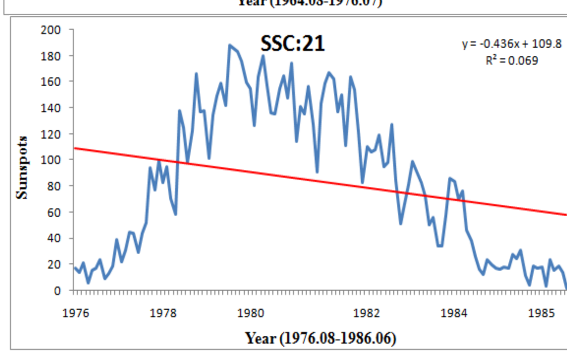
y = -0.217x + 74.53
R² = 0.057
Sunspots
Year (1964.08-1976.07)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y = -0.436x + 109.8
R² = 0.069
Sunspots
Year (1976.08-1986.06)

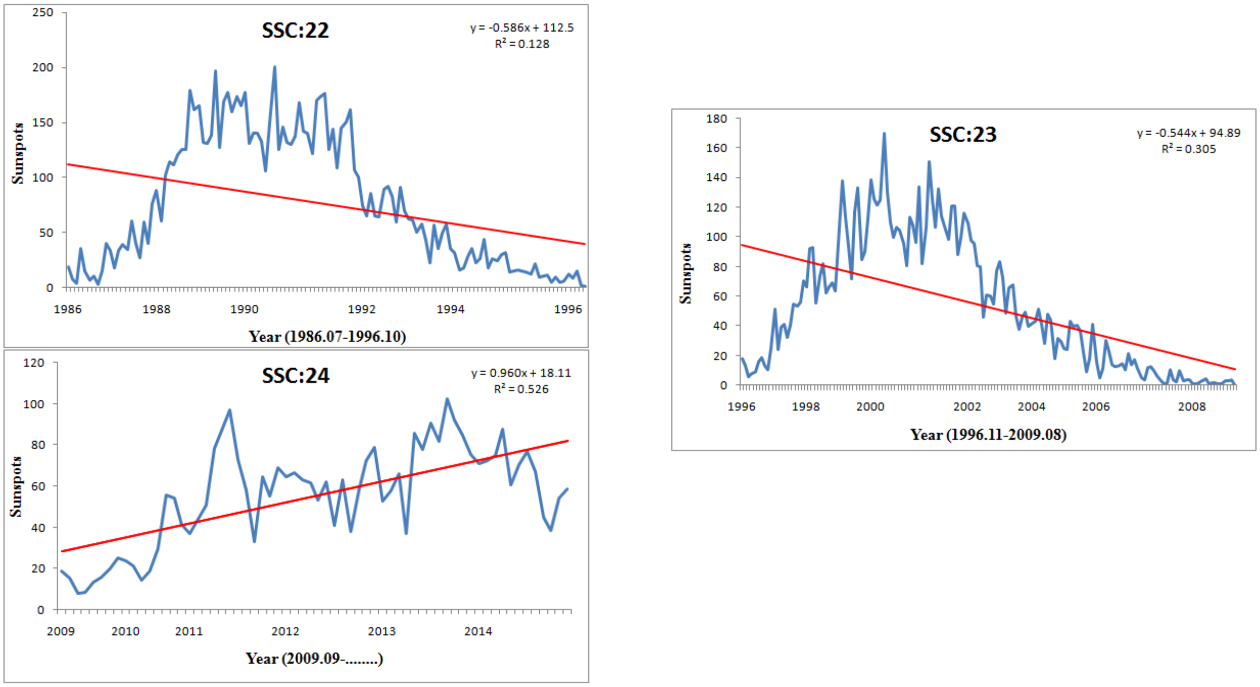
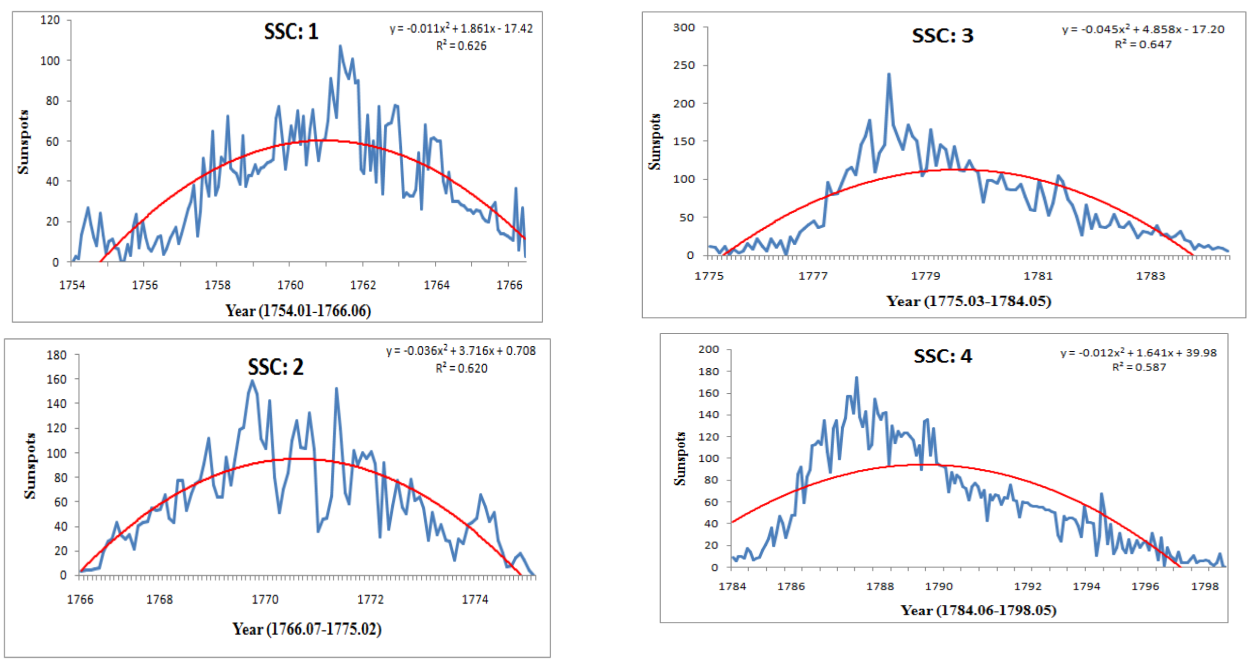
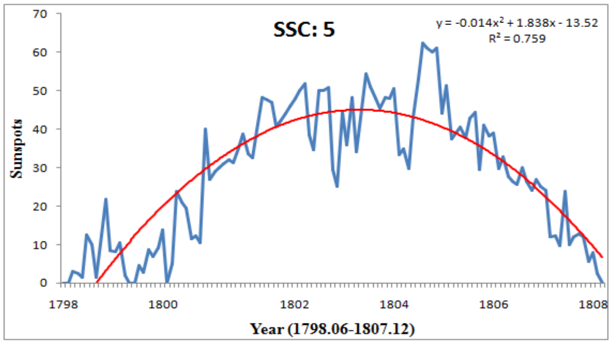


Fig. 1 Time series plots Sunspots cycles with fitted linear trend from 1754-2015

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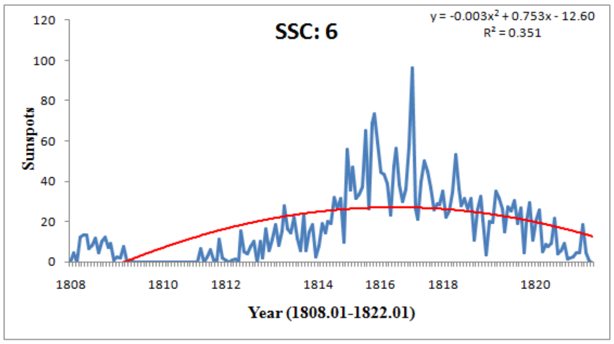
y = -0.014x² + 1.838x - 13.52
R² = 0.759
Sunspots
0
10
20
30
40
50
60
70
1798
1800
1802
1804
1806
1808
Year (1798.06-1807.12)


SSC: 6
y = -0.003x² + 0.753x - 12.60
R² = 0.351
Sunspots
0
20
40
60
80
100
120
1808
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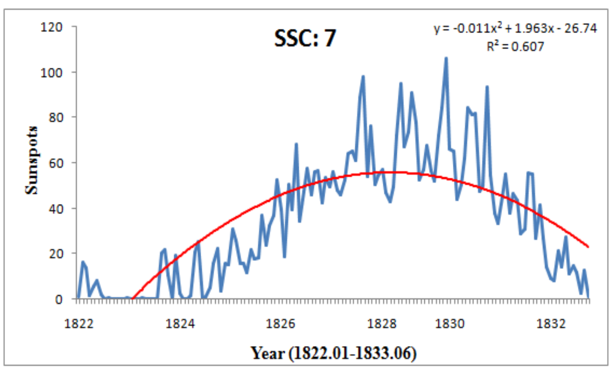
1820
Year (1808.01-1822.01)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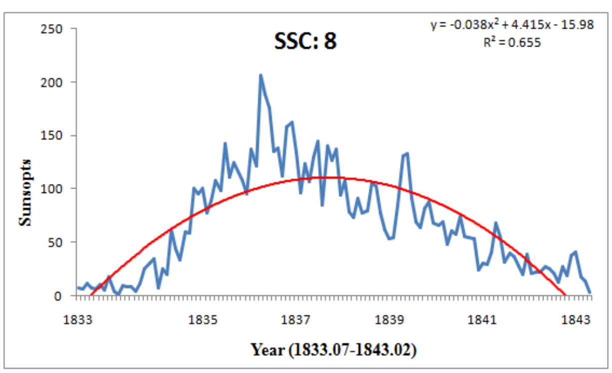
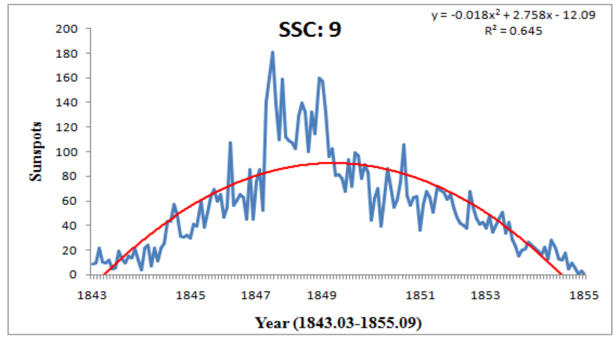
y = -0.011x² + 1.963x - 26.74
R² = 0.607
Sunspots
0
20
40
60
80
100
120
1822
1824
1826
1828
1830
1832
Year (1822.01-1833.06)


SSC: 8
y = -0.038x² + 4.415x - 15.98
R² = 0.655
Sunsopts
0
50
100
150
200
250
1833
1835
1837
1839
1841
1843
Year (1833.07-1843.02)


SSC: 9
y = -0.018x² + 2.758x - 12.09
R² = 0.645
Sunspots
0
20
40
60
80
100
120
140
160
180
200
1843
1845
1847
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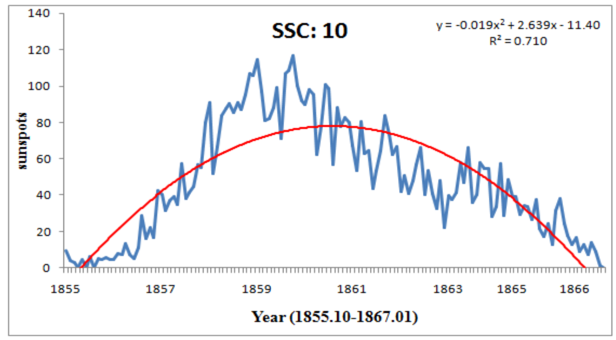
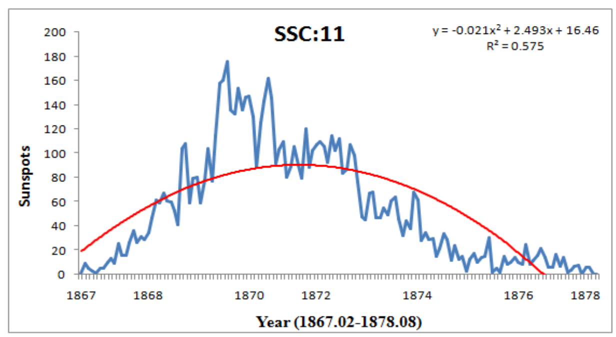
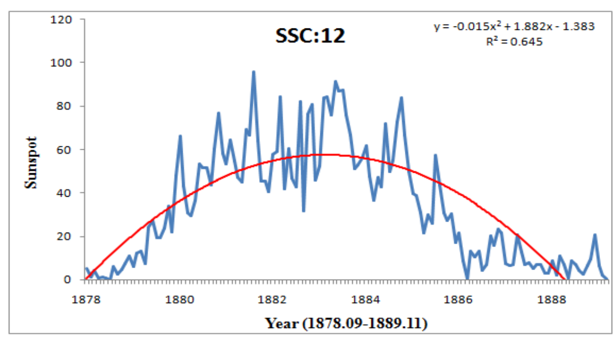
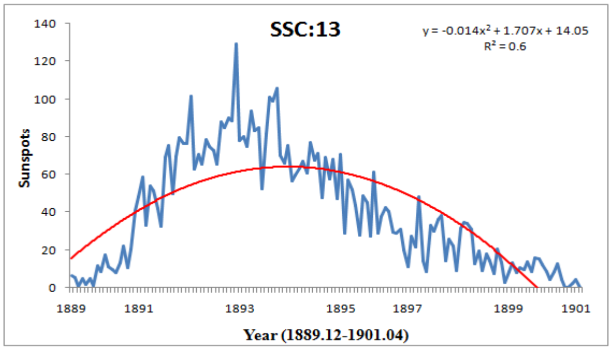
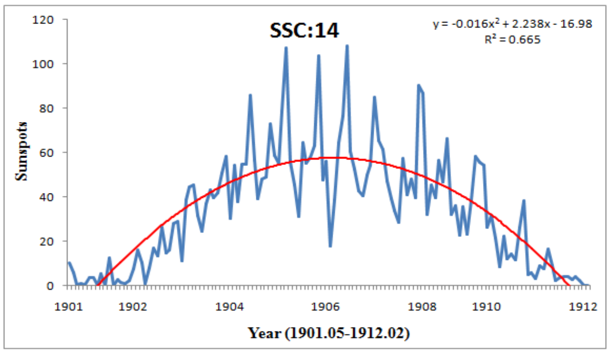
1851
1853
1855
Year (1843.03-1855.09)

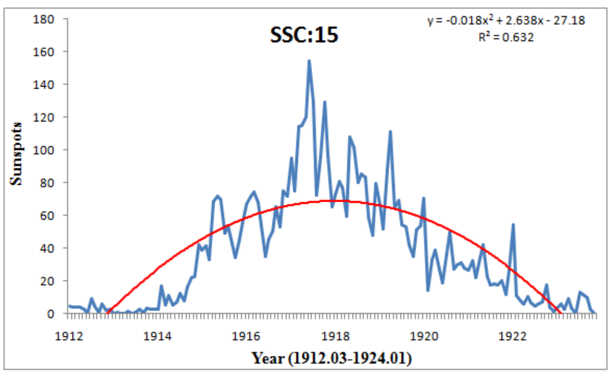

SSC: 10
y = -0.019x² + 2.639x - 11.40
R² = 0.710
sunspots
0
20
40
60
80
100
120
140
1855
1857
1859
1861
1863
1865
1866
Year (1855.10-1867.01)

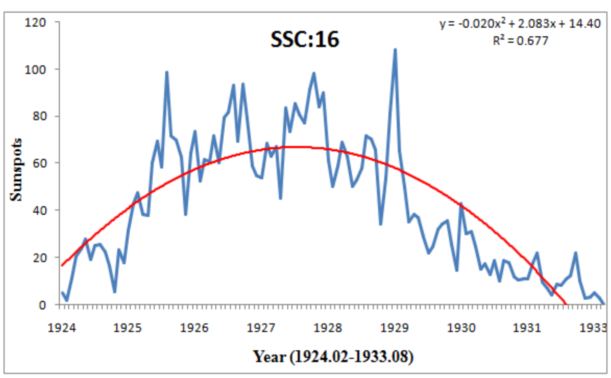

SSC:11
y = -0.021x² + 2.493x + 16.46
R² = 0.575
Sunspots
0
20
40
60
80
100
120
140
160
180
200
1867
1868
1870
1872
1874
1876
1878
Year (1867.02-1878.08)

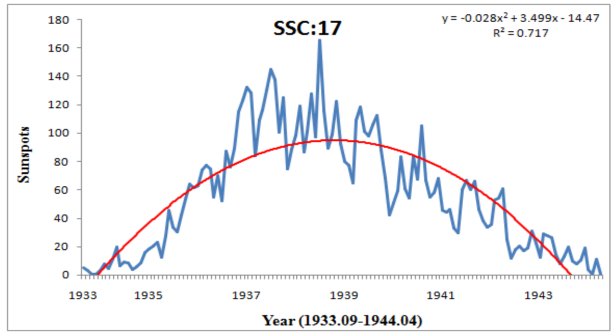

SSC:12
y = -0.015x² + 1.882x - 1.383
R² = 0.645
Sunspot
0
20
40
60
80
100
120
1878
1880
1882
1884
1886
1888
Year (1878.09-1889.11)

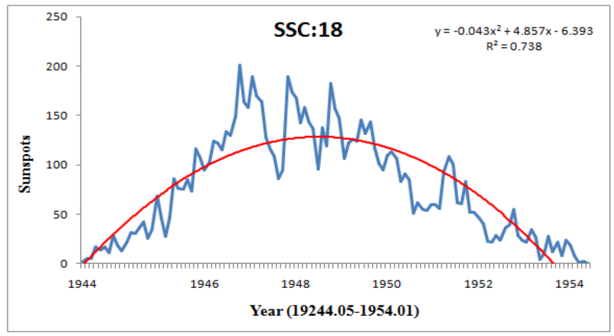

SSC:13
y = -0.014x² + 1.707x + 14.05
R² = 0.6
Sunspots
Year (1889.12-1901.04)

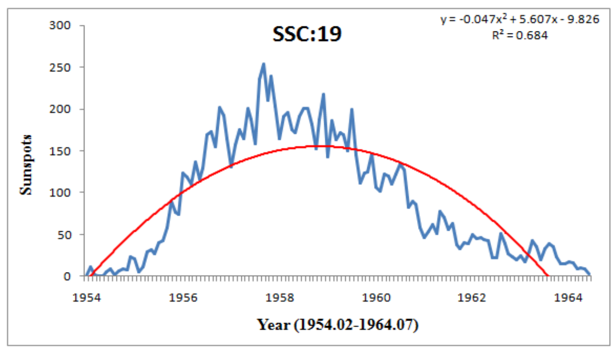

SSC:14
y = -0.016x² + 2.238x - 16.98
R² = 0.665
Sunspots
Year (1901.05-1912.02)

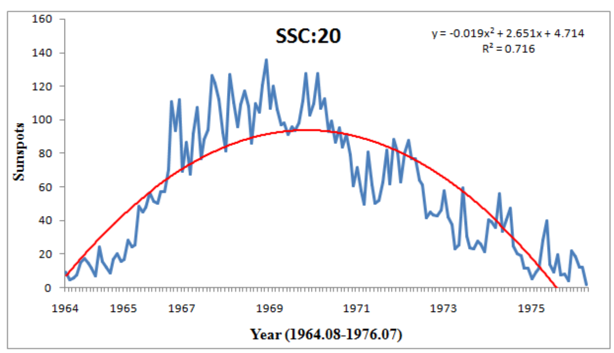

SSC:15
y = -0.018x² + 2.638x - 27.18
R² = 0.632
Sunspots
Year (1912.03-1924.01)


SSC:16
y = -0.020x² + 2.083x + 14.40
R² = 0.677
Sunspots
Year (1924.02-1933.08)


SSC:17
y = -0.028x² + 3.499x - 14.47
R² = 0.717
Sunspots
Year (1933.09-1944.04)


SSC:18
y = -0.043x² + 4.857x - 6.393
R² = 0.738
Sunspots
Year (19244.05-1954.01)


SSC:19
y = -0.047x² + 5.607x - 9.826
R² = 0.684
Sunspots
Year (1954.02-1964.07)


SSC:20
y = -0.019x² + 2.651x + 4.714
R² = 0.716
Sunspots
Year (1964.08-1976.07)

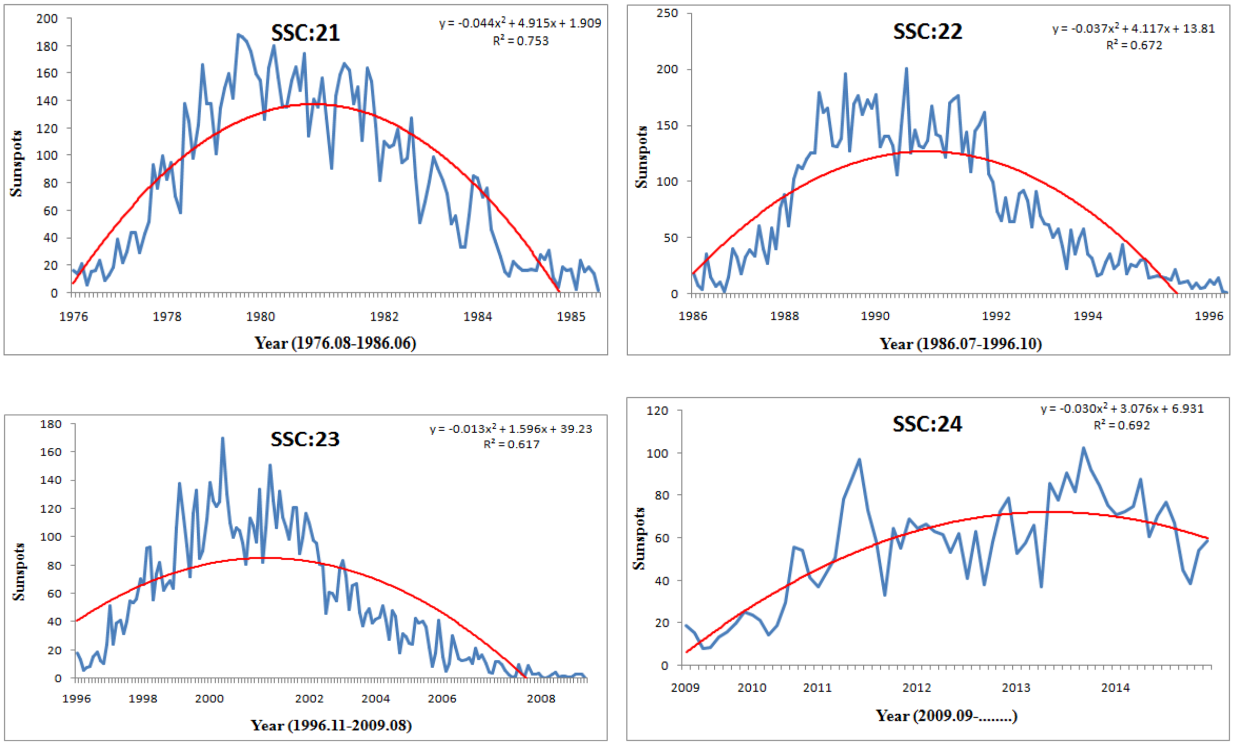


Fig.2 Time series plots of sunspots cycles with fitted nonlinear trend from 1754-2015

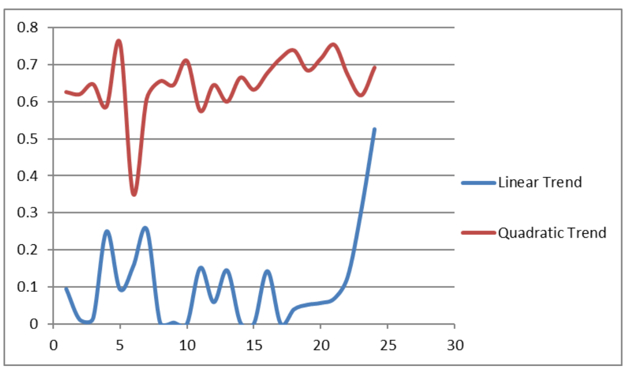


Fig.3 Comparison of the coefficient of determination of linear and quadratic model fitted on 24 sunspot cycles (1754-2015).

Conclusion

Our analysis is shown the quadratic model is best fitted on each sunspot cycle as the nonlinear model. The linear analysis represents the least reliable than the quadratic model according to the goodness-of-fit tests. The Linear trend of sunspot cycle shows long dynamics when increasing. Mostly sunspot cycles indicate the long-term dynamical behavior. The results obtained in this research work shows that both the methods are useful to implement on the solar cycle, but on the comparison nonlinear model are more significant than the linear model.

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